Quantitative Literacy:
Thinking Between the Lines

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Chapter 4:
Personal Finance

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Lesson Plan

- Saving money: The power of compounding
- Borrowing: How much car can you afford?
- Savings for the long term: Build that nest egg
- Credit cards: Paying off consumer debt
- Inflation, taxes, and stocks: Managing your money

Learning Objectives:

- Understand credit cards
- Determine an amount subject to finance charges
- Determine the minimum payment balance formula to find a balance after *t* minimum payments

4.4 Credit cards: Paying off consumer debt

- **Credit card basics**:
- **Amount subject to finance charges**

  = Previous balance – Payment + Purchases

  Where the finance charge is calculated by applying monthly interest rate ($r = \text{APR}/12$) to this amount.
- **New balance**

  = Amount subject to finance charges + Finance charge

- **Example:** Suppose your Visa card calculates finance charges using an APR of 22.8%. Your previous statement showed a balance of $500, in response to which you made a payment of $200. You then bought $400 worth of clothes, which you charged to your Visa card. Find a new balance after one month.

- **Solution:**

- Amount subject to finance charges

  = Previous balance – Payment + Purchases

  = \$500 – \$200 + \$400 = \$700

- Finance charge = $\frac{APR}{12} \times \$700 = \frac{0.228}{12} \times \$700 = \$13.30$

- New Balance

  = Amount subject to finance charges + Finance charge

  = \$700 + \$13.30 = \$713.30

	Previous balance	Payments	Purchases	Finance charge	New balance
Month1	\$500	\$200	\$400	1.9% of \$700 = \$13.30	\$713.30

- **Example:** We have a card with an APR of 24%. The minimum payment is 5% of the balance. Suppose we have a balance of $400 on the card.

  We decide to stop charging and to pay it off by making the minimum payment each month.

  Calculate the new balance after we have made our first minimum payment, and then calculate the minimum payment due for the next month.

- **Solution:**

- 1st minimum payment = 5% of balance = 0.05 × $400 = $20
- Amount subject to finance charges

  = Previous balance – Payment + Purchases

  = $400 – $20 + $0 = $380
- Finance charge $= \frac{APR}{12} \times \$380 = \frac{0.24}{12} \times \$380 = 0.02 \times \$380 = \$7.60$
- New Balance

  = Amount subject to finance charges + Finance charge

  = $380 + $7.60 = $387.60
- The next minimum payment will be 5% of $387.60.

  Minimum payment = 5% of balance = 0.05 × $387.60 = $19.38

- **Minimum payment balance**

Minimum payment balance formula

Balance after t minimum payments

$$= \text{Initial balance} \times [(1+r)(1-m)]^t$$

Where *r* is the monthly rate and *m* is the minimum monthly payment as a percent of the balance.

- **Example:** We have a card with an APR of 20% and a minimum payment that is 4% of the balance. We have a balance of $250 on the card, and we stop charging and pay off that balance by making the minimum payment each month.

Find the balance after two years of payments.

- **Solution:**

- The monthly interest rate: $r = \frac{20\%}{12} = \frac{0.2}{12}$
- The minimum payment = 4% of new balance: m = 0.04
- The initial balance = $250
- The number of payments: t = 2 × 12 = 24 months

Balance after t minimum payments

$$= \text{Initial balance} \times [(1+r)(1-m)]^t$$

$$= \$250 \times \left[\left(1+\frac{0.2}{12}\right)(1-0.04)\right]^{24}$$

$$= \$139.55$$

4.4 Credit cards: Paying off consumer debt

- **Example:** Suppose you have a balance $10,000 on your Visa card, which has an APR of 24%. The card requires a minimum payment of 5% of the balance. You stop charging and begin making only the minimum payment until your balance is below $100.

1. Find a formula that gives your balance after *t* monthly payments.
2. Find your balance after five years of payments.
3. Determine how long it will take to get your balance under $100.
4. Suppose that instead of the minimum payment, you want to make a fixed monthly payment so that your debt is clear in two years. How much do you pay each month?

- **Solution:** 1. The minimum payment as a decimal:

  $m = 0.05$.

- The monthly rate: $r = 0.24/12 = 0.02$
- The initial balance = \$10,000

Balance after t minimum payments

$$= \text{Initial balance} \times [(1 + r)(1 - m)]^t$$
$$= \$10{,}000 \times [(1 + 0.02)(1 - 0.05)]^t$$
$$= \$10{,}000 \times 0.969^t$$

2. Now five years: $t = 5 \times 12 = 60$ months

Balance after 60 months = $\$10{,}000 \times 0.969^{60} = \$1511.56$

- After five years, we still owe over \$1500.

4.4 Credit cards: Paying off consumer debt

3. Determine how long it takes to get the balance down to $100.

- **Method 1 (Using a logarithm):** Solve for t the equation

$$\$100 = \$10{,}000 \times 0.969^t$$

Divide each side of the equation by $10,000:

$$\frac{\$100}{\$10{,}000} = \frac{\$10{,}000}{\$10{,}000} \times 0.969^t$$

$$0.01 = 0.969^t$$

Solve exponential equation using logarithm:

$$A = B^t \text{ is } t = \frac{logA}{logB}$$

Use this formula: $t = \frac{log0.01}{log0.969} = 146.2$ months.

Hence, the balance will be under $100 after 147 monthly payments.

3. Determine how long it takes to get the balance down to $100.

- **Method 2 (Trial and error):** If you want to avoid logarithms, you can solve this problem using trial and error with a calculator. The information in part 2 indicates that it will take some time for the balance to drop below $100.

Try five years or 120 months,

Balance after 120 months = $\$10{,}000 \times 0.969^{120} = \$228.48$.

So we should try large number of months. If you continue in this way, we find the same answer as that obtained for Method 1: the balance drops below $100 at payment 147.

4. Consider your debt as an installment loan:

- Amount borrowed = $10,000
- Monthly interest rate r = APR/12 = 24%/12 = 0.02
- Pay off the loan over 24 years: $t = 24$
- Use the monthly payment formula from section 4.2:

$$\text{Monthly payment} = \frac{\text{Amount borrowed} \times r(1+r)^t}{\left((1+r)^t - 1\right)}$$

$$= \frac{\$10{,}000 \times 0.02 \times 1.02^{24}}{(1.02^{24} - 1)} = \$528.71$$

- So, a payment of $528.71 each month will clear the debt in two years.

Chapter 4 Personal Finance: **Chapter Summary**

- **Savings:** simple interest or compound interest
  - Formulas: simple interest earned
    period interest rate
    balance after *t* periods
    APY
    Present value or Future value
    Number of periods to double
- **Borrowing:** an installment loan
  - Formulas: Monthly payment
    Amount borrowed
  - Fixed-rate mortgage vs. ARM

Chapter 4 Personal Finance: **Chapter Summary**

- Saving for the long term: Build the nest egg (Annuity)
  - Formulas: Balance after *t* deposits
    Needed deposit
    Monthly annuity yield
    Nest egg needed
- Credit cards
  - Formulas: Amount subject to finance charges
    Balance after *t* minimum payments
- Inflation, taxes, and stocks
  - Understand CPI, taxes, DJIA